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(February 4, 2002)

Abstract

Based on gauge invariance, we show that the new threshold phenomenon is discovered in the ϕ radiative decays $\phi \rightarrow \gamma a_0 \rightarrow \gamma \pi^0 \eta$ and $\phi \rightarrow \gamma f_0 \rightarrow \gamma \pi^0 \pi^0$. This enables to conclude that production of the lightest scalar mesons $a_0(980)$ and $f_0(980)$ in these decays is caused by the four-quark transitions, resulting in strong restrictions on the large N_C expansions of the decay amplitudes. The analysis shows that these constraints give new evidences in favor of the four-quark nature of $a_0(980)$ and $f_0(980)$ mesons.

PACS number(s): 12.39.-x, 13.40.Hq, 13.65.+i

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The discovered more than thirty years ago the lightest scalar mesons $a_0(980)$ and $f_0(980)$ became the hard problem for the naive quark-antiquark ($q\bar{q}$) model from the outset. Really, on the one hand the almost exact degeneration of the masses of the isovector $a_0(980)$ and isoscalar $f_0(980)$ states revealed seemingly the structure similar to the structure of the vector ρ and ω mesons, and on the other hand the strong coupling of $f_0(980)$ with the $K\bar{K}$ channel pointed unambiguously to a considerable part of the strange quark pair $s\bar{s}$ in the wave function of $f_0(980)$.

In 1977 R.L. Jaffe noted that in the MIT bag model, which incorporates confinement phenomenologically, there are light four-quark scalar states [1]. He suggested that $a_0(980)$ and $f_0(980)$ might be these states with symbolic structures $a_0^0(980) = (us\bar{u}\bar{s} - ds\bar{d}\bar{s})/\sqrt{2}$ and $f_0(980) = (us\bar{u}\bar{s} + ds\bar{d}\bar{s})/\sqrt{2}$. From that time $a_0(980)$ and $f_0(980)$ resonances came into beloved children of the light quark spectroscopy, see, for example, reviews [2,3].

Ten years later we showed [4] that the study of the radiative decays $\phi \rightarrow \gamma a_0 \rightarrow \gamma\pi\eta$ and $\phi \rightarrow \gamma f_0 \rightarrow \gamma\pi\pi$ can shed light on the problem of $a_0(980)$ and $f_0(980)$ mesons. Over the next ten years the question was considered from different points of view [5–8].

Now these decays have been studied not only theoretically but also experimentally, so that the time is ripe to sum up.

Present time data have already been obtained from Novosibirsk with the detectors SND [9,10] and CMD-2 [11], which give the following branching ratios : $BR(\phi \rightarrow \gamma\pi^0\eta) = (0.88 \pm 0.14 \pm 0.09) \cdot 10^{-4}$ [9] (2000), $BR(\phi \rightarrow \gamma\pi^0\pi^0) = (1.221 \pm 0.098 \pm 0.061) \cdot 10^{-4}$ [10] (2000) and $BR(\phi \rightarrow \gamma\pi^0\eta) = (0.9 \pm 0.24 \pm 0.1) \cdot 10^{-4}$, $BR(\phi \rightarrow \gamma\pi^0\pi^0) = (0.92 \pm 0.08 \pm 0.06) \cdot 10^{-4}$ [11]. DAΦNE also confirms the Novosibirsk results [12].

These data give evidence in favor of the four-quark ($q^2\bar{q}^2$) [1,3,13,14] nature of $a_0(980)$ and $f_0(980)$ mesons [4,6,15]. Note that the isovector $a_0(980)$ meson is produced in the radiative ϕ meson decay as intensively as the well-studied $\eta'(958)$ meson containing $\approx 66\%$ of $s\bar{s}$, responsible for the decay. It is a clear qualitative argument for the presence of the $s\bar{s}$ pair in the isovector $a_0(980)$ state, i.e., for its four-quark nature.

Since the one-loop model $\phi \rightarrow K^+K^- \rightarrow \gamma a_0$ and $\phi \rightarrow K^+K^- \rightarrow \gamma f_0$, suggested at

basing the experimental investigations [4], was used in the data treatment from the outset, the question on the mechanism of the scalar meson production in the ϕ radiative decays was put into the shade. We show below that the present data give the conclusive arguments in favor of the K^+K^- loop mechanism of $a_0(980)$ and $f_0(980)$ mesons production in the ϕ radiative decays. The knowledge of this mechanism allows to conclude that the production of $a_0(980)$ and $f_0(980)$ in the ϕ radiative decays is caused by the four-quark transitions. This constrains the large N_C expansions of the decay amplitudes and gives new impressive evidences in favor of the four-quark nature of $a_0(980)$ and $f_0(980)$.

In Figs. 1 and 2 are shown the SND data on $\phi \rightarrow \gamma\pi^0\eta$ [9] (2000) and $\phi \rightarrow \gamma\pi^0\pi^0$ [10] (2000). The data are described in the following model $\phi \rightarrow (\gamma a_0 + \pi^0\rho) \rightarrow \gamma\pi^0\eta$ and $\phi \rightarrow (\gamma f_0 + \pi^0\rho) \rightarrow \gamma\pi^0\pi^0$, see details in Ref. [15]. It is important for us now nothing but the fact that the $\phi \rightarrow \gamma a_0 \rightarrow \gamma\pi^0\eta$ process dominates everywhere over the region of the $\pi^0\eta$ invariant mass $m_{\pi^0\eta} = m$ and the $\phi \rightarrow \gamma f_0 \rightarrow \gamma\pi^0\pi^0$ process dominates in the resonance region of the $\pi^0\pi^0$ system, the $\pi^0\pi^0$ invariant mass $m_{\pi^0\pi^0} = m > 670$ MeV [16].

The resonance mass spectrum is of the form [17]

$$S_R(m) = d\Gamma(\phi \rightarrow \gamma R \rightarrow \gamma ab, m)/dm \\ = \frac{2}{\pi} \frac{m^2 \Gamma(\phi \rightarrow \gamma R, m) \Gamma(R \rightarrow ab, m)}{|D_R(m)|^2} = \frac{4|g_R(m)|^2 \omega(m) p_{ab}(m)}{3(4\pi)^3 m_\phi^2} \left| \frac{g_{Rab}}{D_R(m)} \right|^2, \quad (1)$$

where $R = a_0$ or f_0 and $ab = \pi^0\eta$ or $\pi^0\pi^0$ respectively, $\omega(m) = (m_\phi^2 - m^2)/2m_\phi$ is the photon energy in the ϕ meson rest frame, $p_{ab}(m)$ is the modulus of the a or b particle momentum in the a and b mass center frame, g_{Rab} is the coupling constant, $g_{f_0\pi^+\pi^-} = g_{f_0\pi^0\pi^0}/\sqrt{2}$, $D_R(m)$ is the R resonance propagator the form of which everywhere over the m region can be find in [4,18,19], $g_R(m)$ is the invariant amplitude that describes the vertex of the $\phi(p) \rightarrow \gamma(k)R(q)$ transition with $q^2 = m^2$. This is precisely the function which is the subject of our investigation.

By gauge invariance, the transition amplitude is proportional to the electromagnetic field strength tensor $F_{\mu\nu}$ (in our case to the electric field in the ϕ meson rest frame):

$$A[\phi(p) \rightarrow \gamma(k)R(q)] = G_R(m) [p_\mu e_\nu(\phi) - p_\nu e_\mu(\phi)] [k_\mu e_\nu(\gamma) - k_\nu e_\mu(\gamma)], \quad (2)$$

where $e(\phi)$ and $e(\gamma)$ are the ϕ meson and γ quantum polarization four-vectors, $G_R(m)$ is the invariant amplitude free from kinematical singularities. Since there are no charge particles or particles with magnetic moments in the process, there is no pole in $G_R(m)$. Consequently, the function

$$g_R(m) = -2(pk)G_R(m) = -2\omega(m)m_\phi G_R(m) \quad (3)$$

is proportional to $\omega(m)$ (at least!) in the soft photon region.

To describe the experimental spectra in Figs. 1 and 2, the function $|g_R(m)|^2$ should be smooth (almost constant) in the range $m \leq 0.99$ GeV, see Eq. (1). Stopping the function $(\omega(m))^2$ at $\omega(990 \text{ MeV}) = 29 \text{ MeV}$ with the help of the form-factor $1/[1 + (R\omega(m))^2]$ requires $R \approx 100 \text{ GeV}^{-1}$. It seems to be incredible to explain the formation of such a huge radius in hadron physics. Based on the large, by hadron physics standard, $R \approx 10 \text{ GeV}^{-1}$, one can obtain an effective maximum of the mass spectrum under discussion only near 900 MeV. To exemplify this trouble let us consider the contribution of the isolated R resonance: $g_R(m) = -2\omega(m)m_\phi G_R(m_R)$. Let also the mass and the width of the R resonance equal 980 MeV and 60 MeV, then $S_R(920 \text{ MeV}) : S_R(950 \text{ MeV}) : S_R(970 \text{ MeV}) : S_R(980 \text{ MeV}) = 3 : 2.7 : 1.8 : 1$.

So stopping the $g_R(m)$ function is the crucial point in understanding the mechanism of the production of $a_0(980)$ and $f_0(980)$ resonances in the ϕ radiative decays.

The K^+K^- loop model $\phi \rightarrow K^+K^- \rightarrow \gamma R$ [4] solves this problem in the elegant way: the fine threshold phenomenon is discovered, see Fig. 3, where the universal in K^+K^- loop model function $|g(m)|^2 = |g_R(m)/g_{Rab}|^2$ is shown [20].

To demonstrate the threshold character of this effect we present Fig. 4 and Fig. 5 in which the function $|g(m)|^2$ is shown in the case of K^+ meson mass is 25 MeV and 50 MeV less than in reality.

One can see from Figs. 4 and 5 that the function $|g(m)|^2$ is suppressed by the $(\omega(m))^2$ law in the region 950-1020 MeV and 900-1020 MeV respectively [21].

In the mass spectrum this suppression is increased by one more power of $\omega(m)$, see Eq.

(1), so that we cannot see the resonance in the region 980-995 MeV. The maximum in the spectrum is effectively shifted to the region 935-950 MeV and 880-900 MeV respectively.

In truth this means that $a_0(980)$ and $f_0(980)$ resonances are seen in the radiative decays of ϕ meson owing to the K^+K^- intermediate state, otherwise the maxima in the spectra would be shifted to 900 MeV.

So the mechanism of production of $a_0(980)$ and $f_0(980)$ mesons in the ϕ radiative decays is established.

Both the real part and the imaginary one of the $\phi \rightarrow \gamma R$ amplitude are caused by the K^+K^- intermediate state. The imaginary part is caused by the real K^+K^- intermediate state while the real part is caused by the virtual compact K^+K^- intermediate state, i.e., we are dealing here with the four-quark transition [22]. Needless to say, radiative four-quark transitions can happen between two $q\bar{q}$ states as well as between $q\bar{q}$ and $q^2\bar{q}^2$ states but their intensities depend strongly on a type of the transitions. A radiative four-quark transition between two $q\bar{q}$ states requires creation and annihilation of an additional $q\bar{q}$ pair, i.e., such a transition is forbidden according to the Okuba-Zweig-Izuka (OZI) rule, while a radiative four-quark transition between $q\bar{q}$ and $q^2\bar{q}^2$ states requires only creation of an additional $q\bar{q}$ pair, i.e., such a transition is allowed according to the OZI rule.

Let us consider this problem from the large N_C expansion standpoint. Let us begin with the $q\bar{q}$ model.

In the two-quark model $a_0^0(980) = (u\bar{u} - d\bar{d})/\sqrt{2}$, the large N_C expansion of the $\phi = s\bar{s} \rightarrow \gamma a_0(980)$ amplitude starts with the OZI forbidden transition of the $1/N_C$ order ($s\bar{s}$ annihilation and $u\bar{u}$, $d\bar{d}$ creation). But its weight is bound to be small, because this term does not contain the K^+K^- intermediate state emerging only in the next to leading term of the $(1/N_C)^2$ (!) order for creation and annihilation of additional $q\bar{q}$ pairs. It will be recalled that $\phi \rightarrow \gamma \eta'(958)$ transition does not require creation of an additional $q\bar{q}$ pair at all (the OZI superallowed transition) and has the $(N_C)^0$ order.

In the two-quark model $f_0(980) = (u\bar{u} + d\bar{d})/\sqrt{2}$, which involves the a_0 - f_0 mass degeneration, the large N_C expansion of the $\phi = s\bar{s} \rightarrow \gamma f_0(980)$ amplitude starts also with the OZI

forbidden transition of the $1/N_C$ order, whose weight also is bound to be small, because this term does not contain the K^+K^- intermediate state emerging only in the next to leading term of the $(1/N_C)^2$ order.

In the two-quark model $f_0(980) = s\bar{s}$, which has the serious trouble with the a_0 - f_0 mass degeneration, the $(N_C)^0$ order transition without creation of an additional $q\bar{q}$ pair (similar in this regard to $\phi \rightarrow \gamma\eta'(958)$) is bound to have a small weight in the large N_C expansion of the $\phi = s\bar{s} \rightarrow \gamma f_0(980)$ amplitude, because this term does not contain the K^+K^- intermediate state emerging only in the next to leading term of the $1/N_C$ order, i.e., in the OZI forbidden transition. Emphasize that the mechanism without creation and annihilation of the additional $u\bar{u}$ pair cannot explain the $S_{f_0}(m)$ spectrum because it does not contain the K^+K^- intermediate state!

But if $a_0^0(980)$ and $f_0(980)$ mesons are compact $K\bar{K}$ states $(u\bar{s}s\bar{u} - d\bar{s}s\bar{d})/\sqrt{2}$ and $(u\bar{s}s\bar{u} + d\bar{s}s\bar{d})/\sqrt{2}$ respectively, i.e., four-quark states similar to states of the MIT-bag model [23], the large N_C expansions of the $\phi = s\bar{s} \rightarrow \gamma a_0(980)$ and $\phi = s\bar{s} \rightarrow \gamma f_0(980)$ amplitudes start with the OZI allowed transitions of the $(1/N_C)^{-1/2}$ order, which require only creation the additional $u\bar{u}$ pair for the K^+K^- intermediate state. It will be recalled that a OZI allowed hadronic decay amplitude, for example, the $\rho \rightarrow \pi\pi$ amplitude, has the $(1/N_C)^{-1/2}$ order.

In summary the fine threshold phenomenon is discovered, which is to say that the K^+K^- loop mechanism of the $a_0(980)$ and $f_0(980)$ scalar meson production in the ϕ radiative decays is established at a physical level of proof. The case is rarest in hadron physics. This production mechanism is the four-quark transition what constrains the large N_C expansion of the $\phi \rightarrow \gamma a_0(980)$ and $\phi \rightarrow \gamma f_0(980)$ amplitudes and gives the new strong (if not crucial) evidences in favor of the four-quark nature of $a_0(980)$ and $f_0(980)$ mesons.

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- [20] The forms of $g_R(m)$ and $g(m) = g_R(m)/g_{Rab}$ everywhere over the m region are in Refs. [4] and [19] respectively.
- [21] The actual absence of a background at a soft photon energy region $\omega(m) > 112$ MeV ($m > 900$ MeV) owes to gauge invariance also.
- [22] It will be recalled that the imaginary part of every hadronic amplitude describes a multi-quark transition.
- [23] In the case of the $K\bar{K}$ bound states with the binding energy close to 20 MeV , i.e., in the extended molecule case, the contribution of the virtual intermediate K^+K^- states in the K^+K^- loop is suppressed by the momentum distribution in the molecule, and the real part of the loop amplitudes are negligible [7]. It leads to the branching ratios much less than the experimental ones. In addition, the $S_R(m)$ spectra in the $K\bar{K}$ molecule case are much narrower than the experimental ones, see the behavior of the imaginary part contribution in Fig. 3.

FIGURES

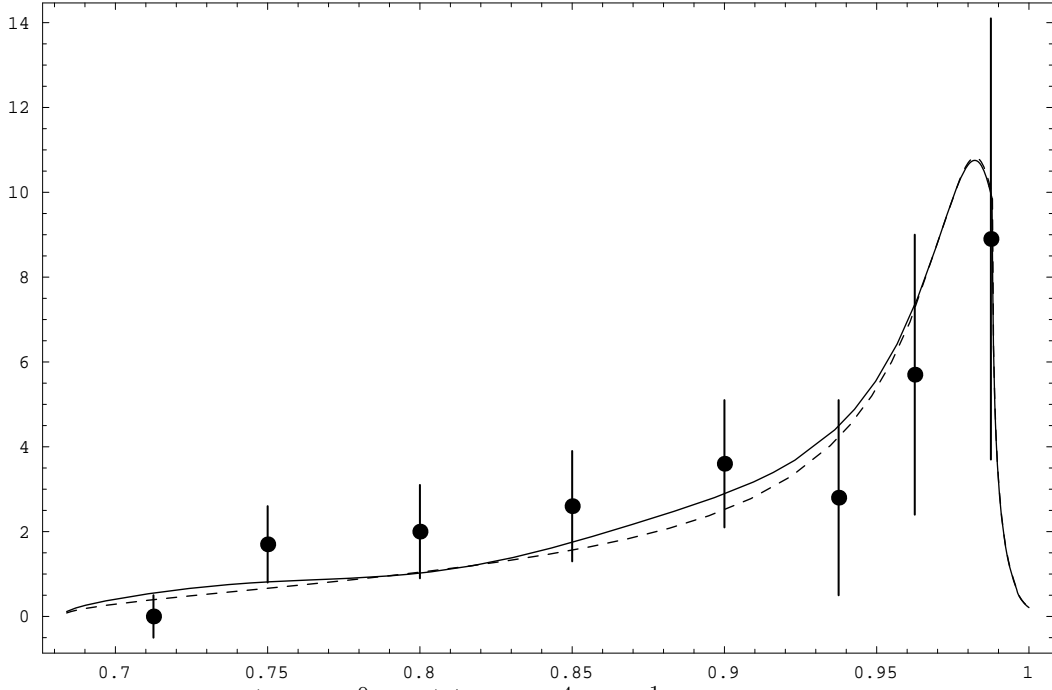


FIG. 1. Fitting of $dBR(\phi \rightarrow \gamma\pi^0\eta, m)/dm \times 10^4 \text{GeV}^{-1}$ is shown with the solid line, the signal contribution $\phi \rightarrow \gamma a_0 \rightarrow \gamma\pi^0\eta$ is shown with the dashed line.

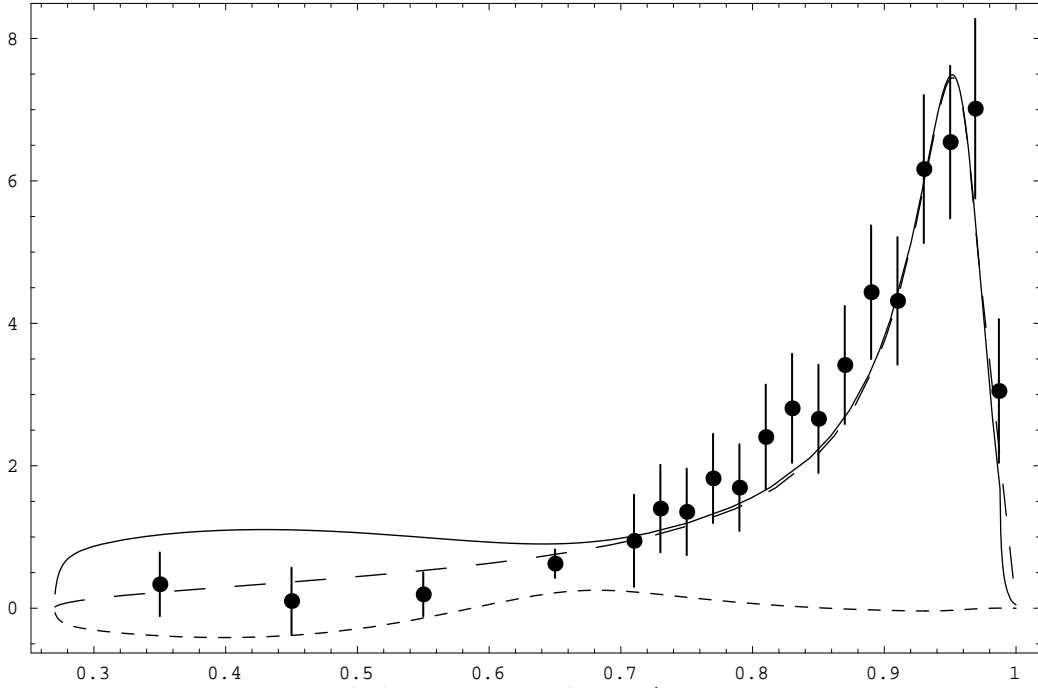


FIG. 2. Fitting of $dBR(\phi \rightarrow \gamma\pi^0\pi^0, m)/dm \times 10^4 \text{GeV}^{-1}$ is shown with the solid line, the signal contribution $\phi \rightarrow \gamma f_0 \rightarrow \gamma\pi^0\pi^0$ is shown with the dashed line. The dots show the interference.

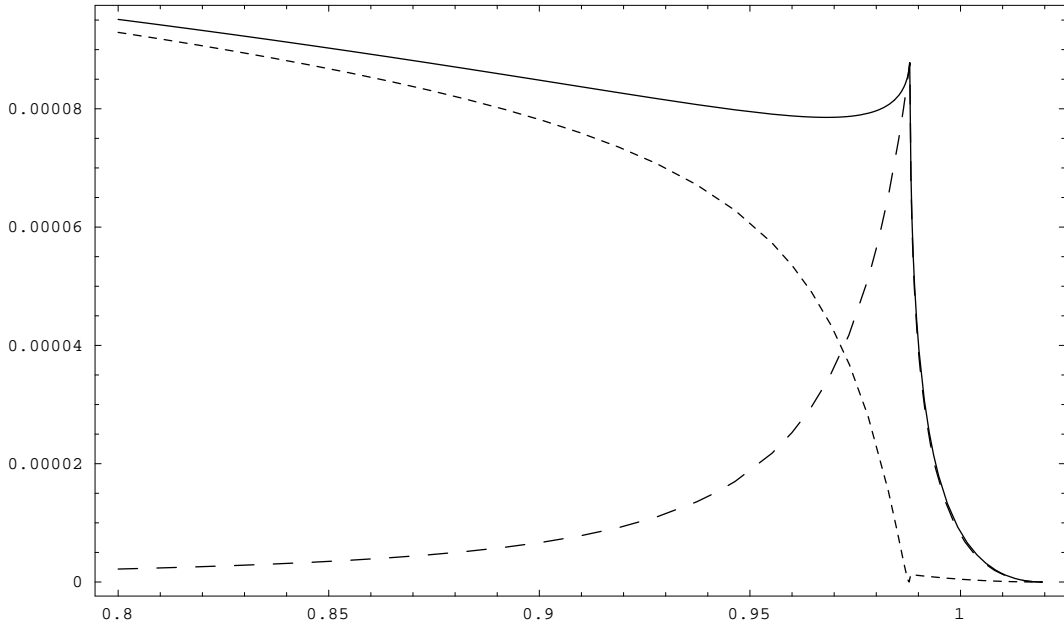


FIG. 3. The function $|g(m)|^2$ is drawn with the solid line. The contribution of the imaginary part is drawn with the dashed line. The contribution of the real part is drawn with the dotted line.

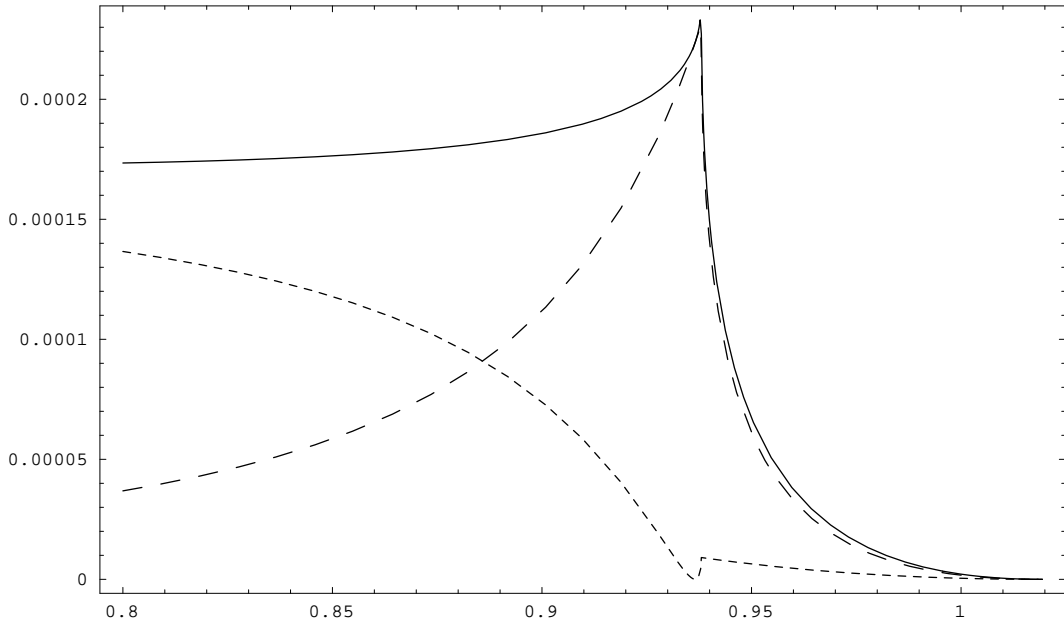


FIG. 4. The function $|g(m)|^2$ for $m_{K^+} = 469$ MeV is drawn with the solid line. The contributions of the imaginary and real parts are drawn with the dashed and dotted lines respectively.

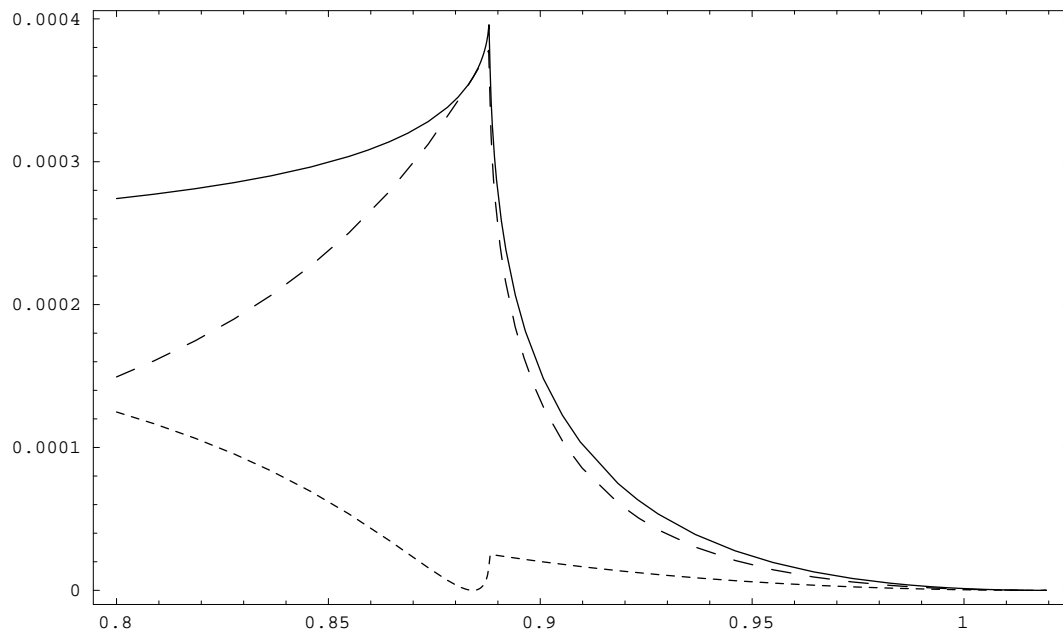


FIG. 5. The function $|g(m)|^2$ for $m_{K^+} = 444$ MeV is drawn with the solid line. The contributions of the imaginary and real parts are drawn with the dashed and dotted lines respectively.